Inventory management 1

Yuji Yamamoto

PPU426 – HT 2018

MÄLARDALEN UNIVERSITY SWEDEN

Today’s topic

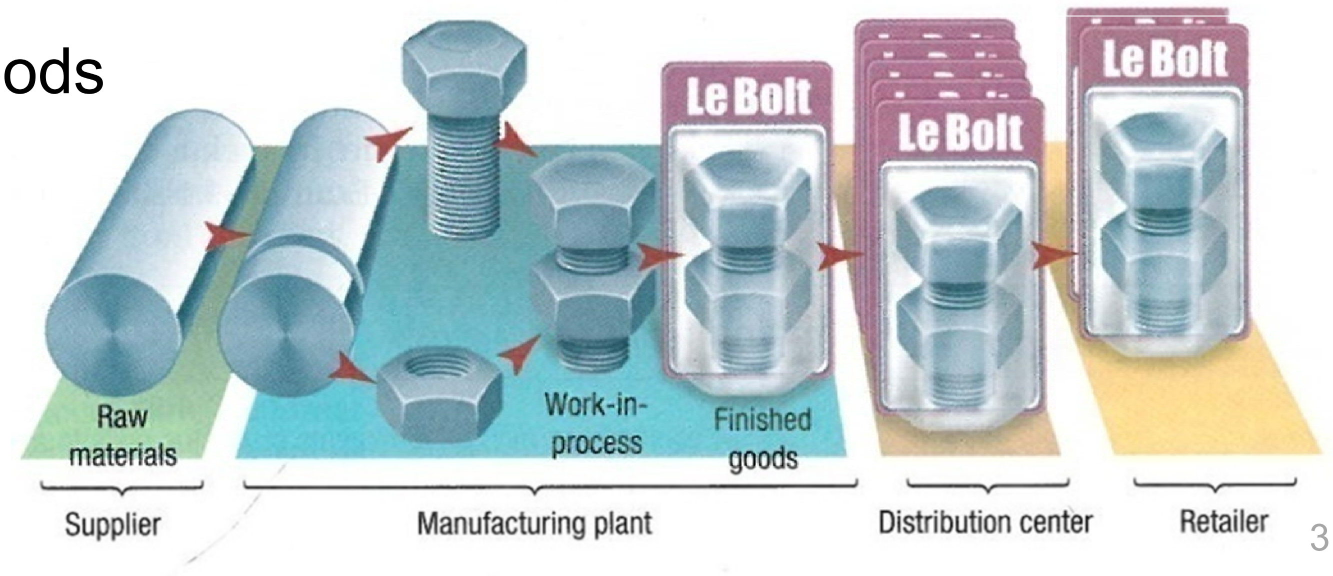
- What is inventory?
- Inventory profile
- How to decide how much to order?
- How to decide when to order?

What is inventory?

- a stock of materials

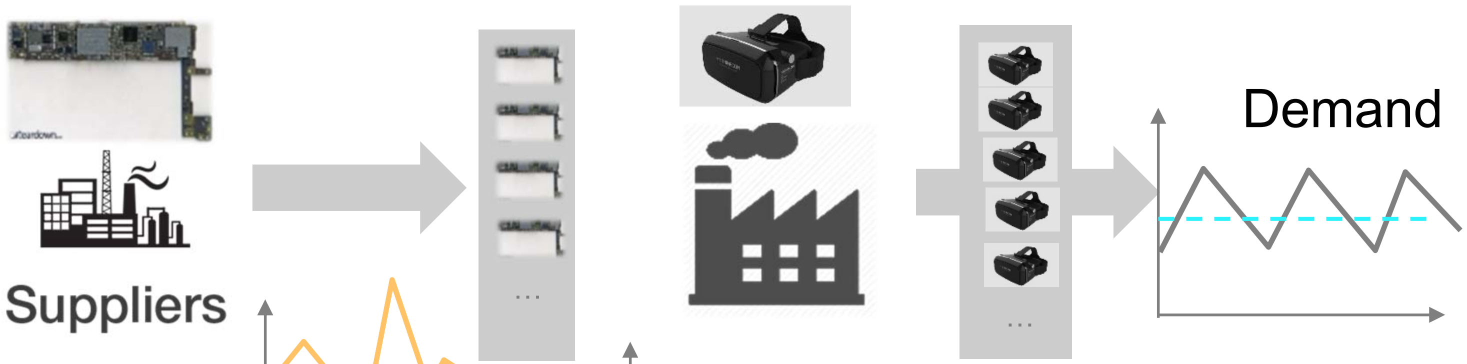
Types of inventory:

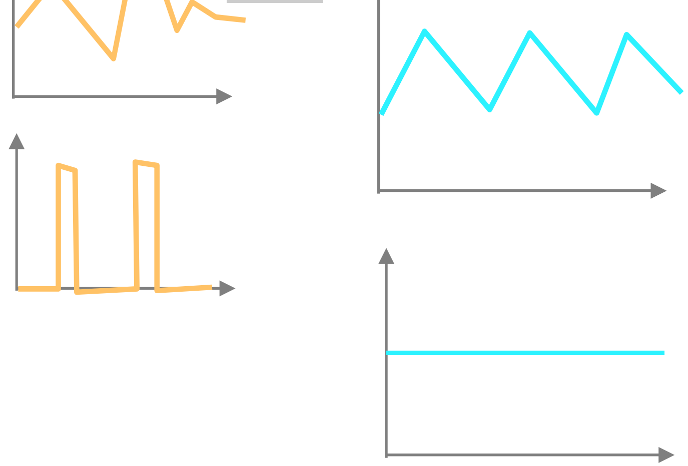
- Raw material
- Work in process (WIP)
- Finished goods



Why having inventory?

Major reason: absorb the mismatch btw supply and demand





Why having inventory?

Absorbing mismatch in various situations:

- Smoothing the production
- Insurance against uncertainties
- Secure sales opportunities

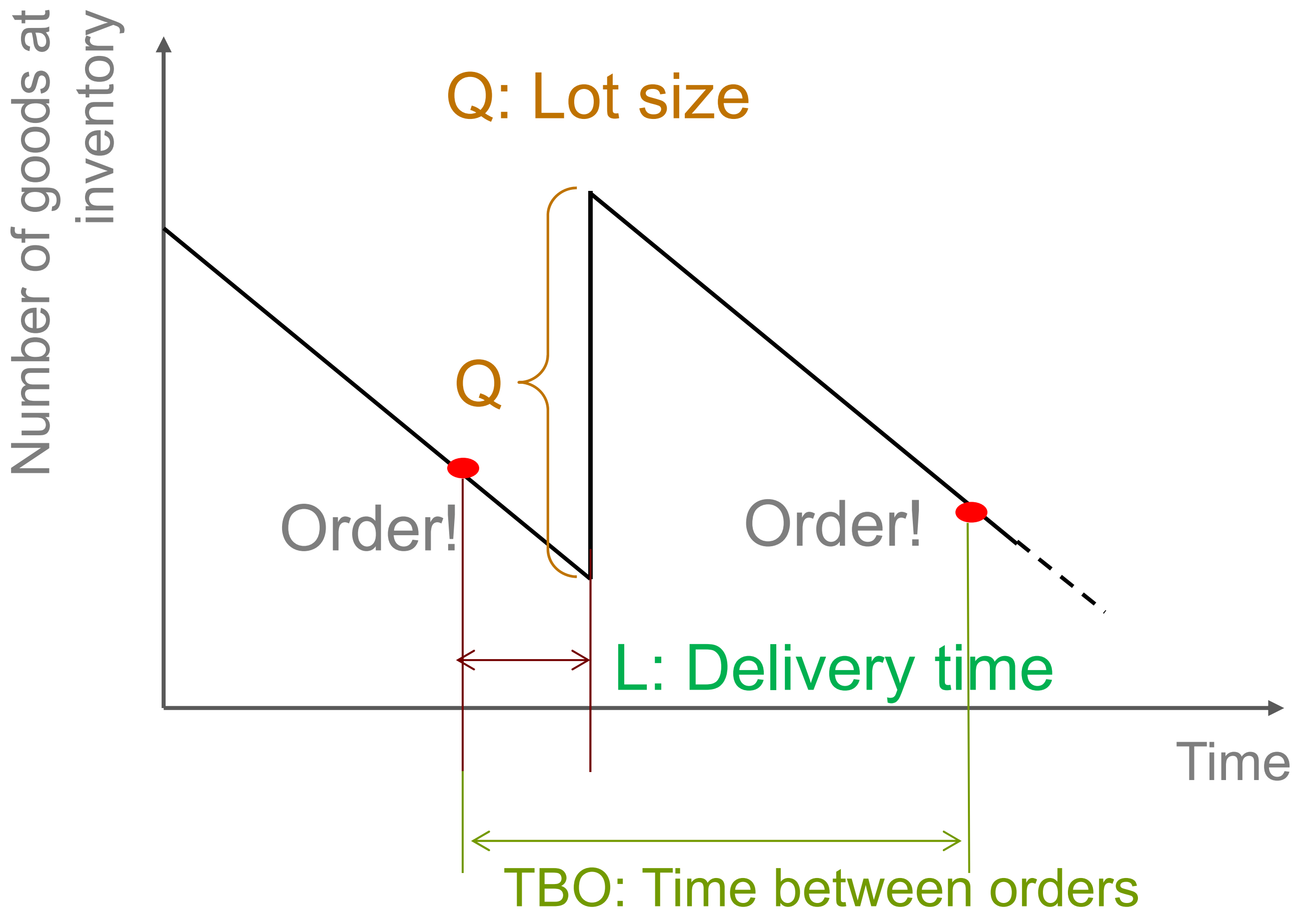
Drawbacks of having large inventory?

- Tying working capital
- Deterioration
- Obsoleting
- Taking space
- Staff cost
- Hiding problem (TPS, Lean)

➡ Enough to secure operations but not too much!

How much inventory to have?

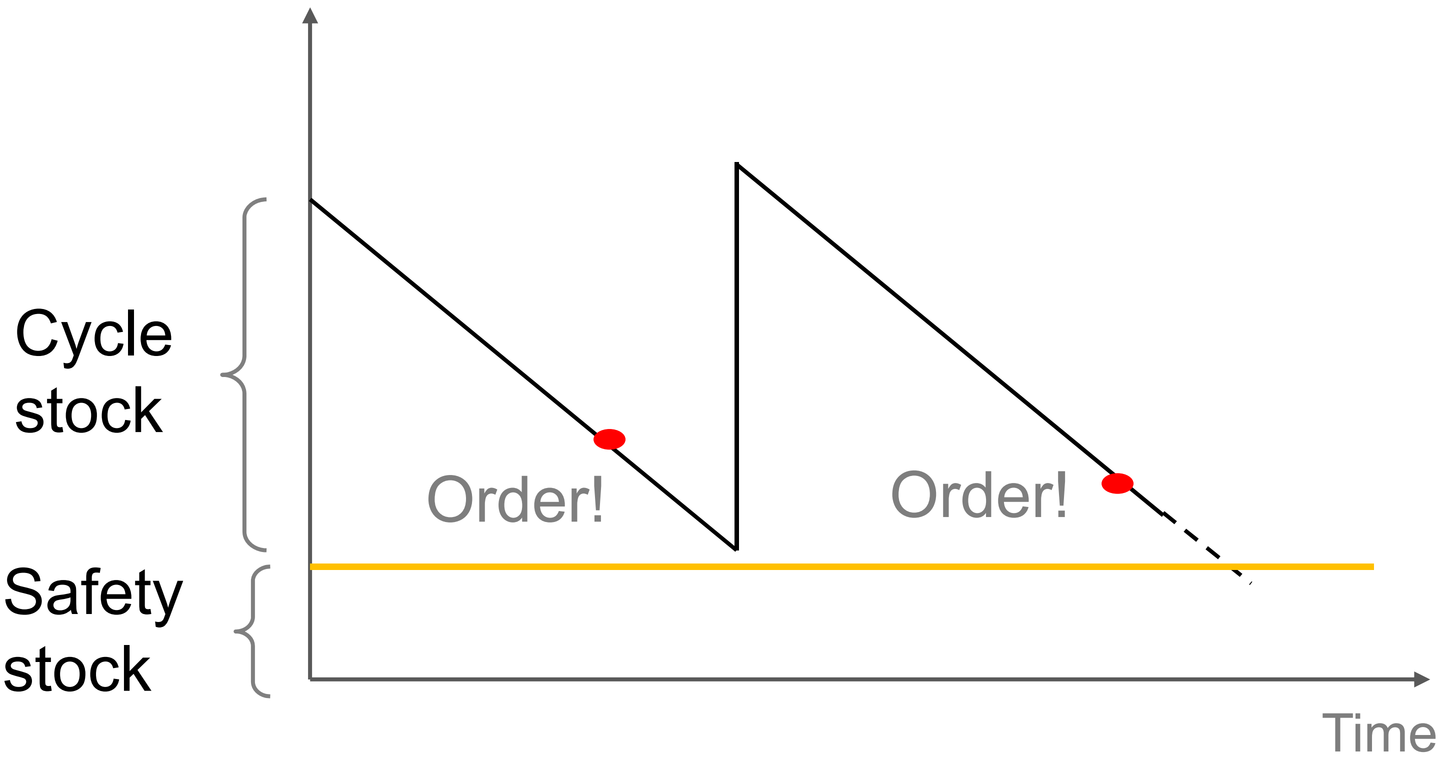
Inventory profile



50pcs/month
(600pcs/year)

How much inventory to have?

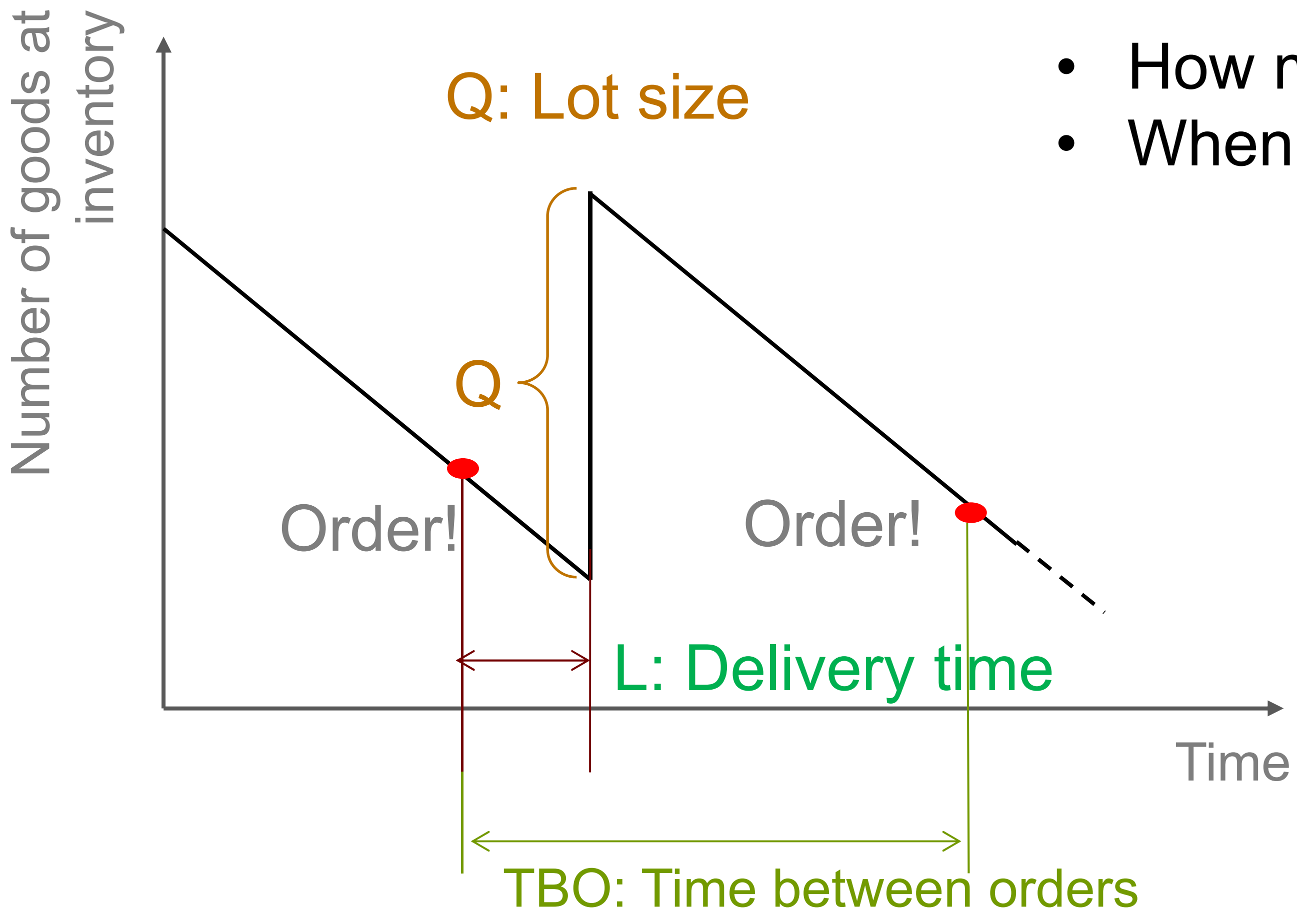
Inventory profile



50pcs/month
(600pcs/year)

How much inventory to have?

Inventory profile

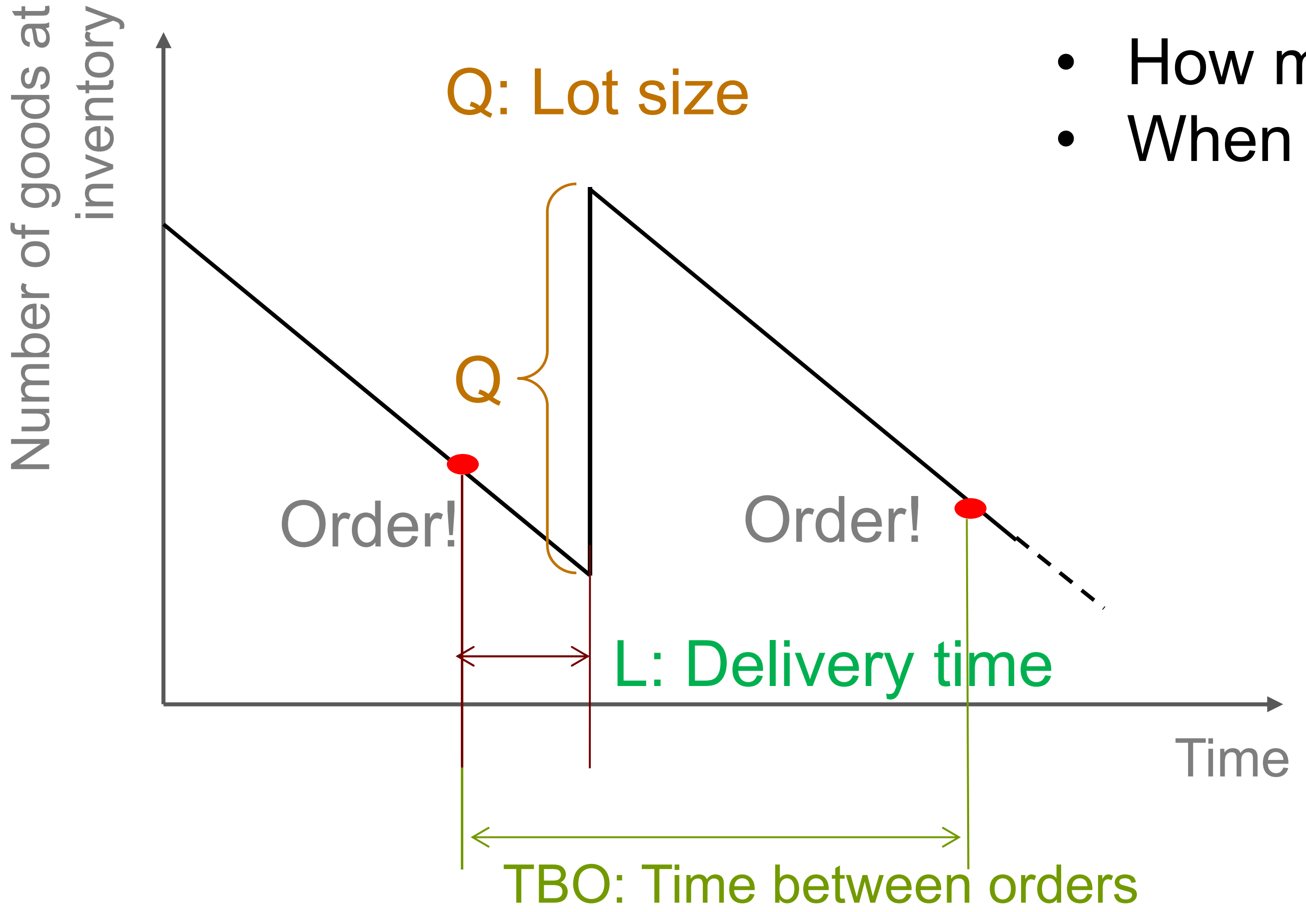


- How much to order?
- When to order?

50pcs/month
(600pcs/year)

How much inventory to have?

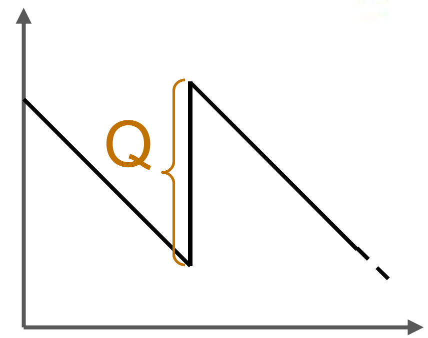
Inventory profile



- How much to order?
- When to order?

50pcs/month
(600pcs/year)

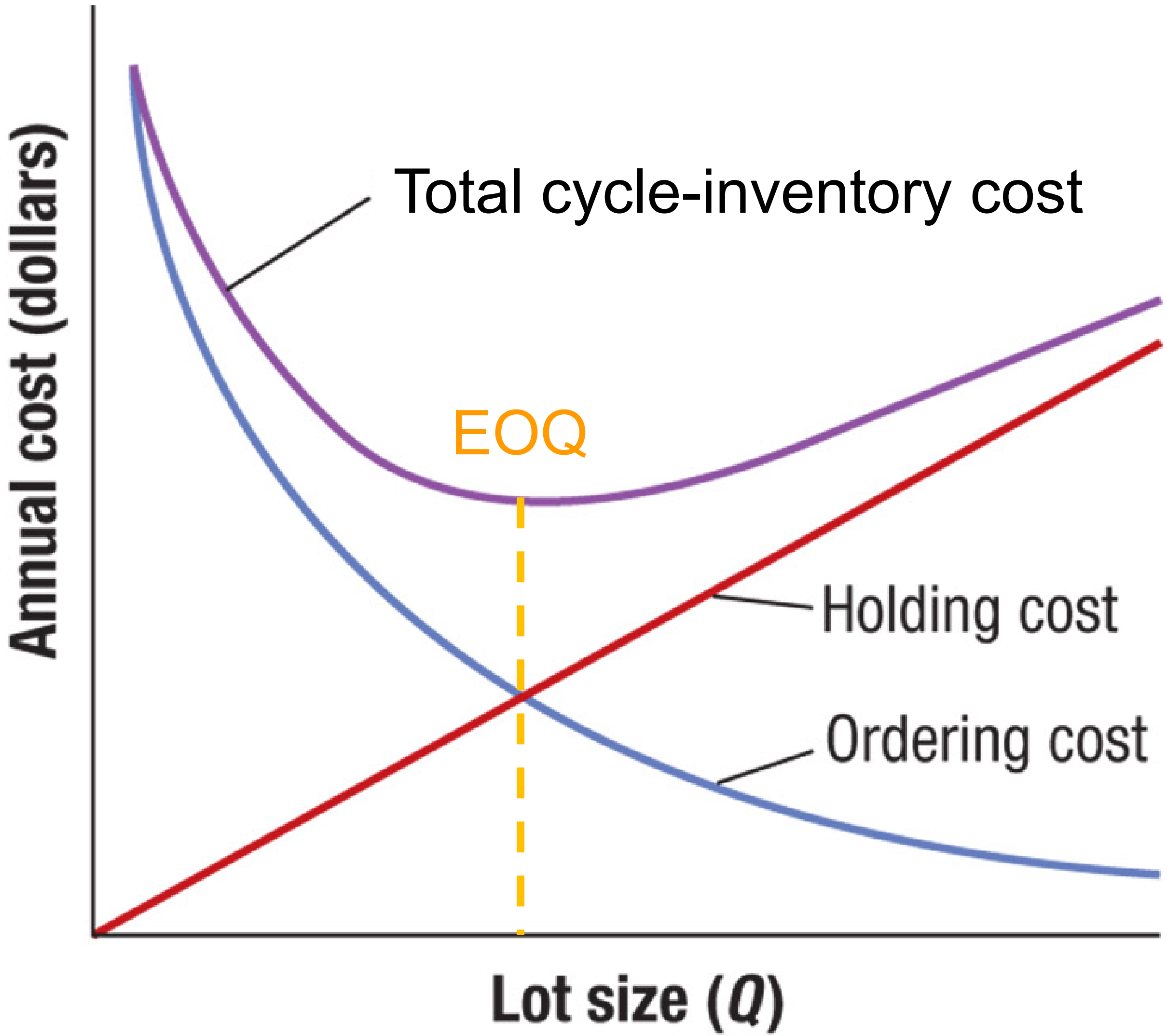
How much to order?



	+	-
Q ↗	• Less frequent order • Less extern transport • More discount • …	• More space • Risk of obsolete • More staff • …
Q ↘	• Less space • Less risk of obsolete • Less staff • …	• More frequent order • More extern transport • Less discount • …

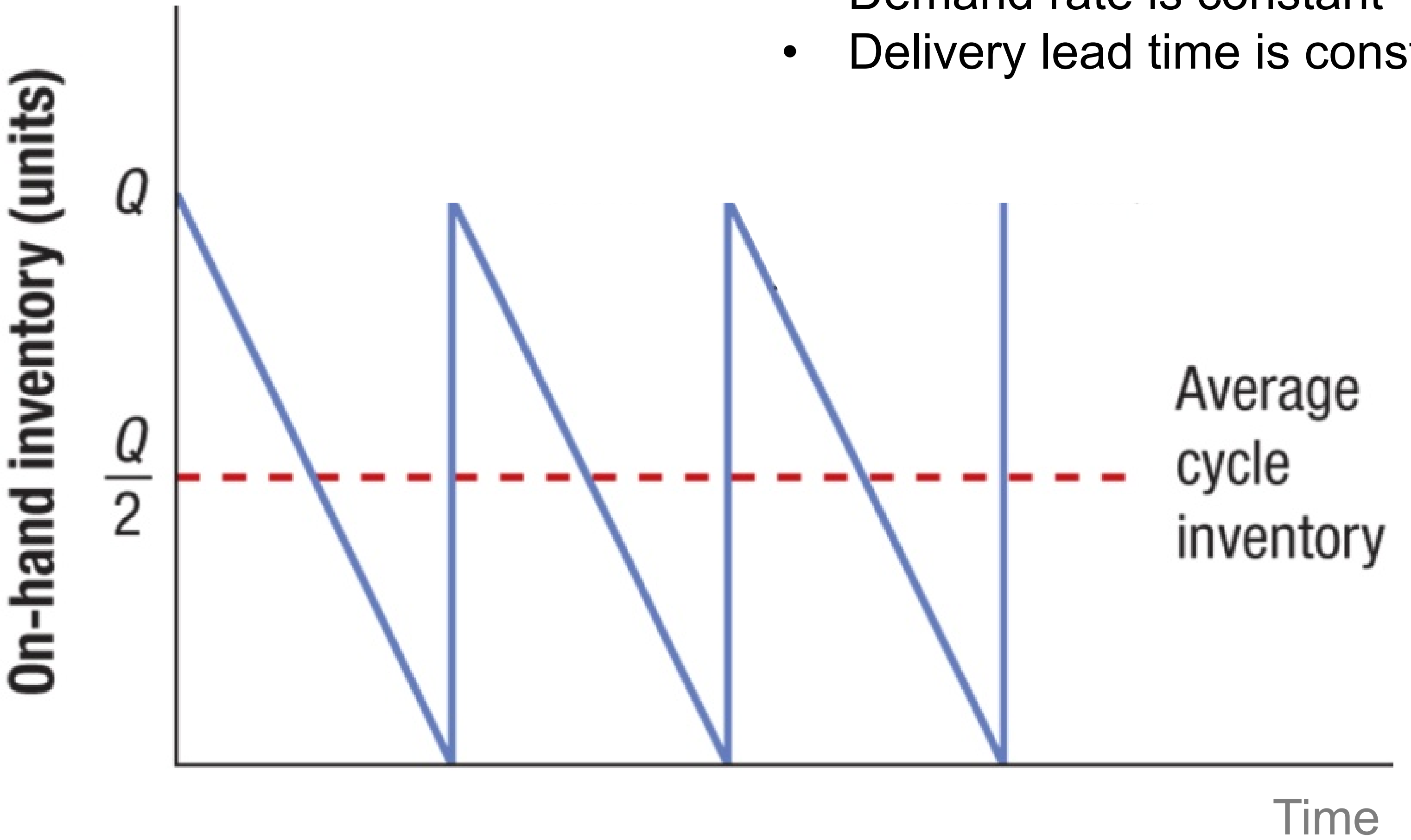
MÄLARDALEN UNIVERSITY
SWEDEN

Economic Order Quantity (EOQ)



EOQ: Assumption

- Demand rate is constant
- Delivery lead time is constant

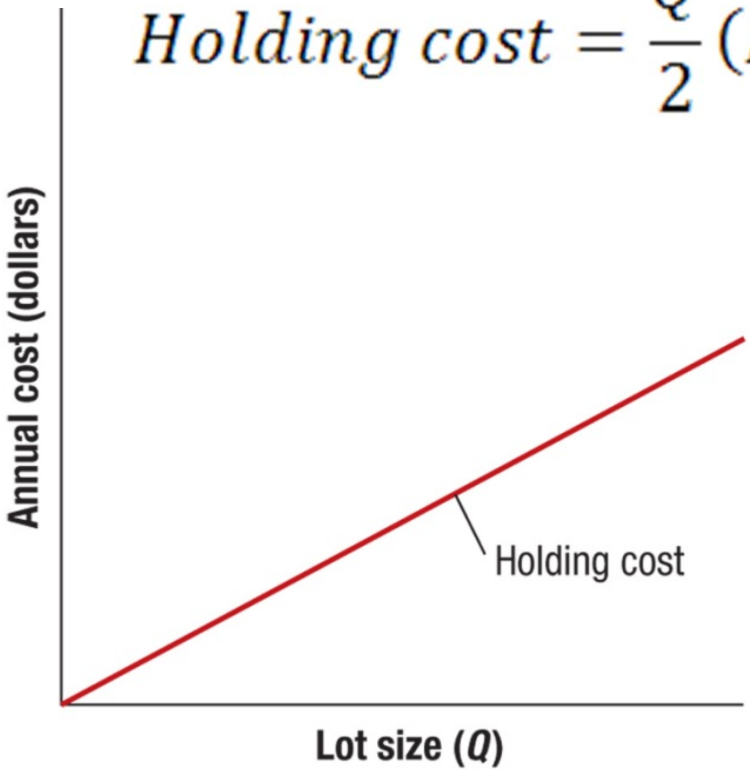


Note: Discount (incl. external transportation cost) is not included in this model

MÄLARDALEN UNIVERSITY SWEDEN

Holding cost

$$Holding\ cost = \frac{Q}{2}(H)$$



(a) Annual holding cost

Q: Lot size

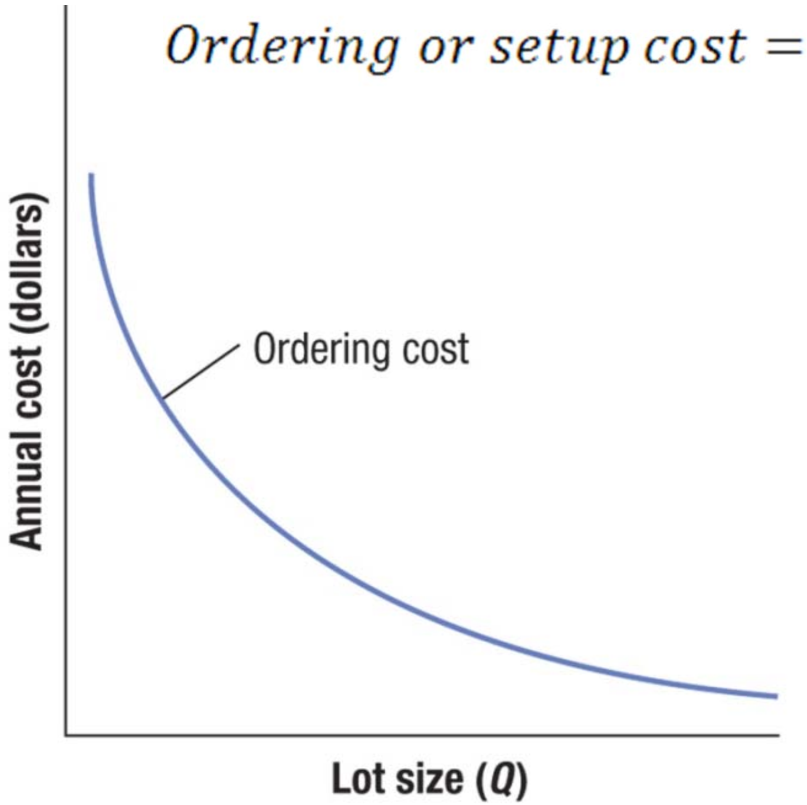
H: Holding cost of one unit during a year (often percentage of unit value)

Holding cost includes…

- Lost of capital (cost of sleeping money)
- Storage and handling
- Tax and insurance
- Theft
- Obsolete
- Deteriorate

Ordering cost

$$Ordering\ or\ setup\ cost = \frac{D}{Q}(S)$$



(b) Annual ordering cost

D: Annual demand
S: Ordering cost per order

Ordering cost includes…

- Ordering (administrative) cost
- Setup cost

MÄLARDALEN UNIVERSITY
SWEDEN

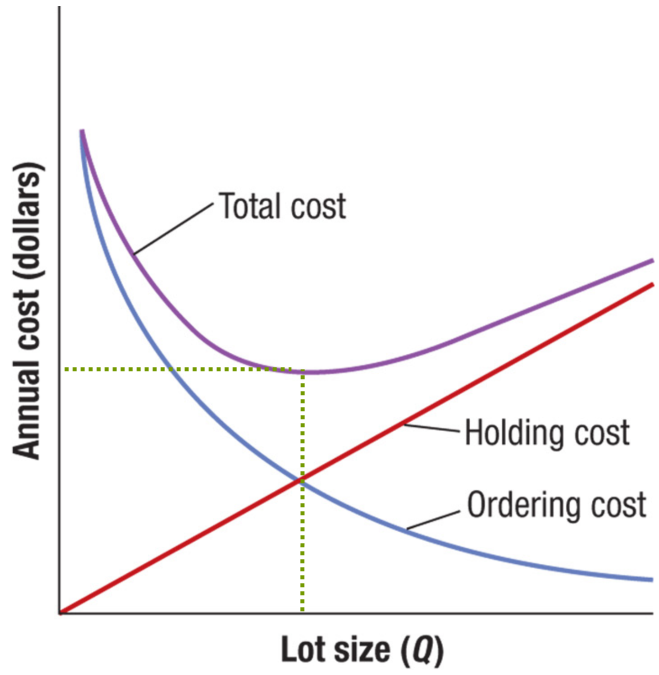
EOQ calculation

Total annual cycle-inventory cost

= Annual holding cost + Annual ordering cost

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

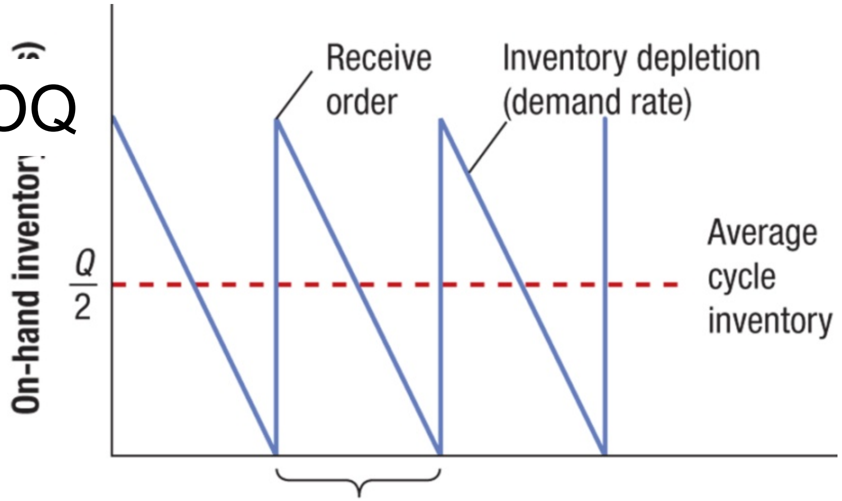
$$EOQ = \sqrt{\frac{2DS}{H}}$$



(c) Total annual cycle-inventory cost

TBO at EOQ

EOQ



?

$$TBO_{EOQ} = \frac{EOQ}{D}$$

Example:

D = 1000
EOQ = 100

TBO_{EOQ} = 100/1000
=1/10

EXAMPLE 1

EOQ, Cost, TBO

- You sell BF 18 units per week
- You pay for the supplier BF, $60 per unit
- Cost of placing an order $45 per order
- Annual holding cost of BF is 25% of BF's value
- The shop is open 52 weeks per year
- Current lot size is 390

Question:

1) What is the current total inventory cost (annual)?
2) EOQ? Total cost at EOQ?
3) TBO_{EOQ} ?

a)

Solution We begin by computing the annual demand and holding cost as

$$D = (18 \text{ units/week})(52 \text{ weeks/year}) = 936 \text{ units}$$

$$H = 0.25(\$60/\text{unit}) = \$15$$

Thus the annual cost is

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

$$= \frac{390}{2}(\$15) + \frac{936}{390}(\$45) = \$2925 + \$108 = \$3033$$

b) For the birdfeeders in Example 12.1, calculate the EOQ and its total cost. How frequently will orders be placed if the EOQ is used?

Solution Using the formulas for EOQ and annual cost, we get

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(936)(45)}{15}} = 74.94, \qquad \text{or 75 units}$$

$$C = \frac{75}{2}(\$15) + \frac{936}{75}(\$45) = \$562 + \$562 = \$1124$$

The EOQ is 75 units and the cost \$1124. This cost is much less than the \$3033 cost of the current policy of placing 390-unit orders.

The time between orders (TBO) when the EOQ is used is given below both in months and in weeks (assuming 52 business weeks per year):

$$TBO_{EOQ} = \frac{EOQ}{D}(12 \text{ months/year}) = \frac{75}{936}(12) = 0.96 \text{ month}$$

$$TBO_{EOQ} = \frac{EOQ}{D}(52 \text{ weeks/year}) = \frac{75}{936}(52) = 4.17 \text{ weeks}$$

$$TBO_{EOQ} = \frac{EOQ}{D} 365 \text{ days/y} = \frac{75}{936} 365 = 29.25 \text{ days}$$

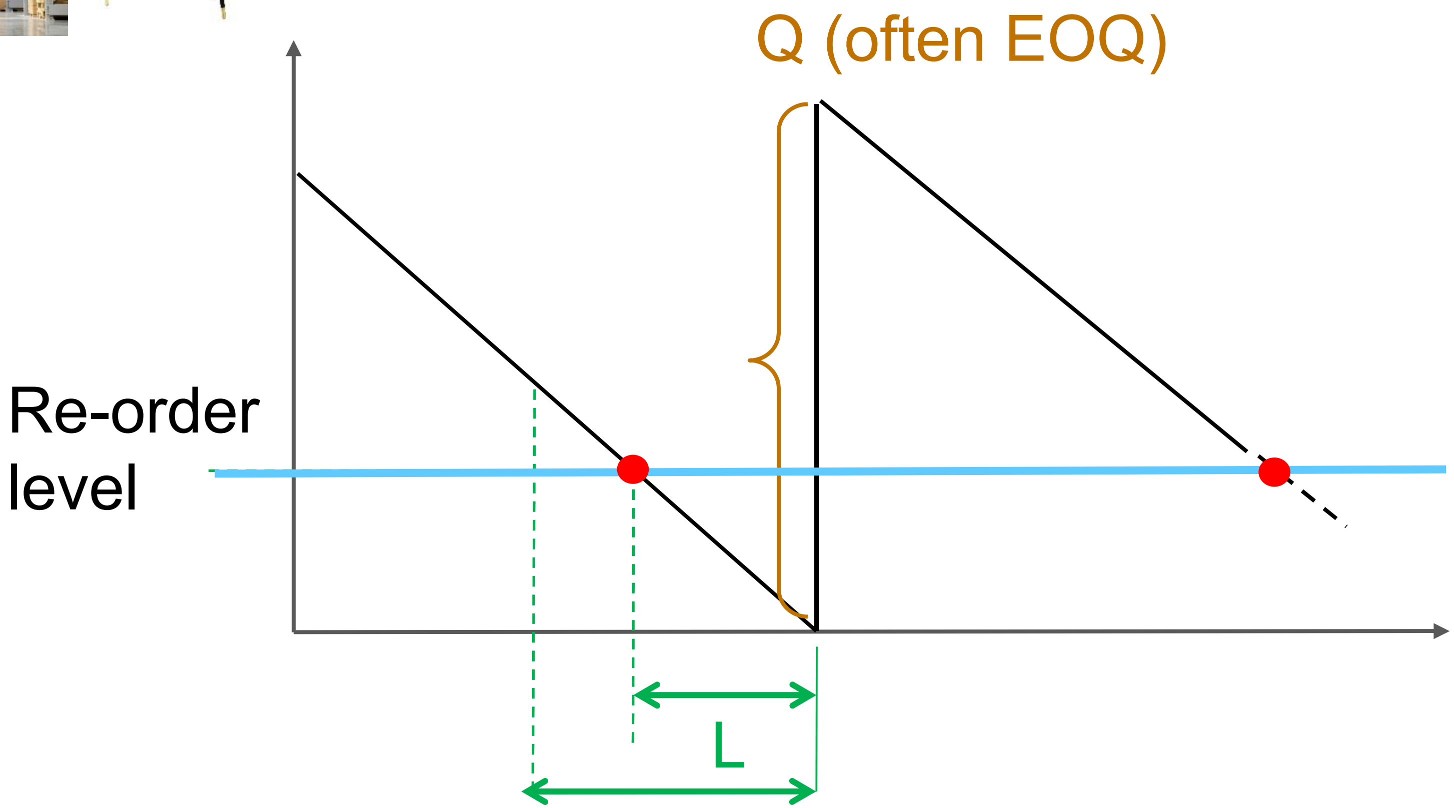
When to place order?

Three well known inventory control systems:

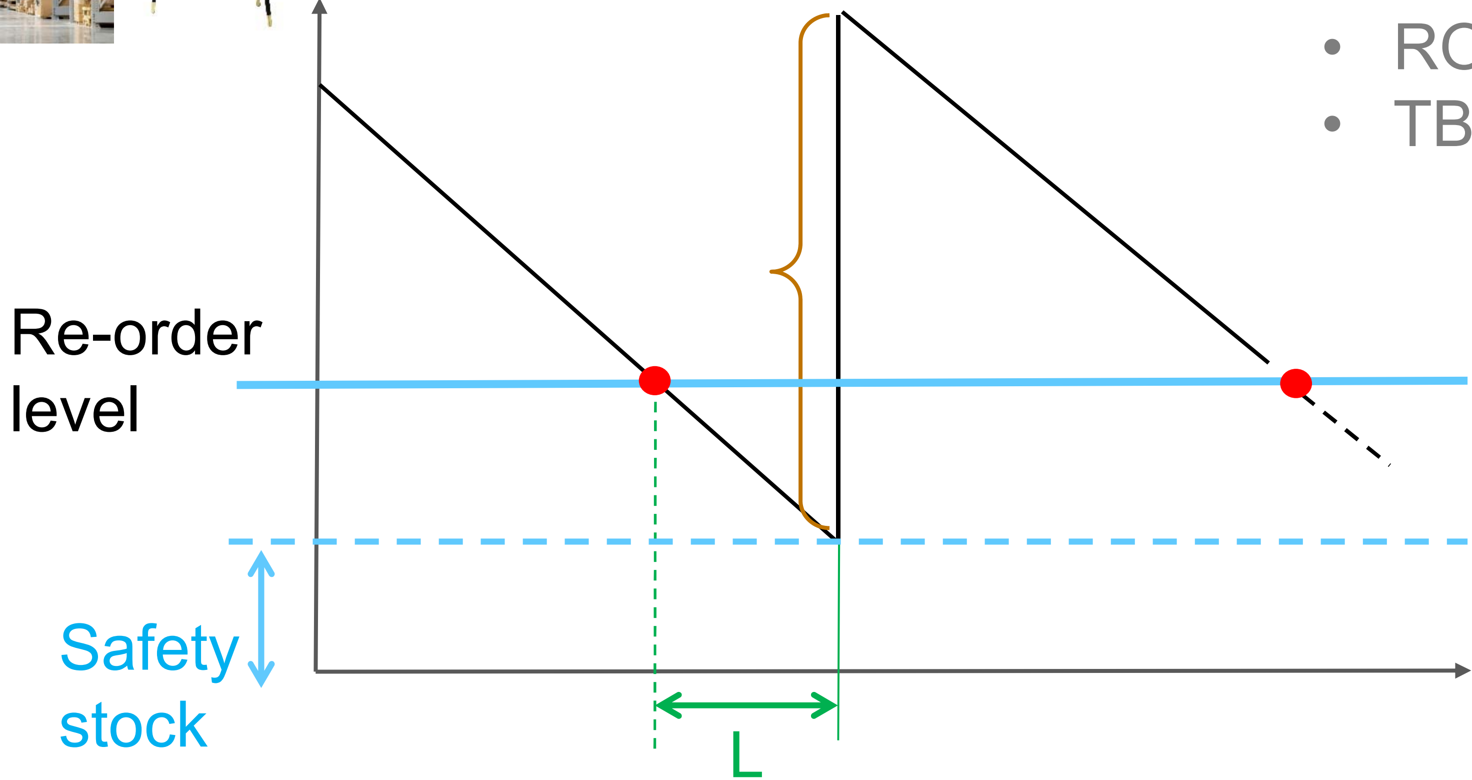
- Continuous review (Q) system
- Periodic review (P) system
- Two bin

MÄLARDALEN UNIVERSITY SWEDEN

Continuous review (Q) system



Continuous review (Q) system



- Q: Const.
- ROL: Const.
- TBO: Flex

MÄLARDALEN UNIVERSITY SWEDEN

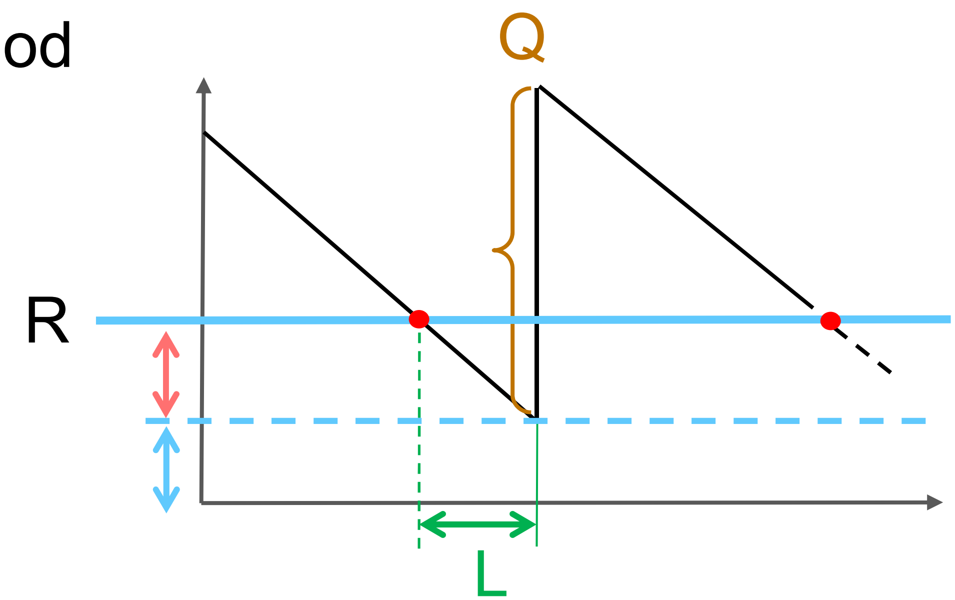
Re-order point

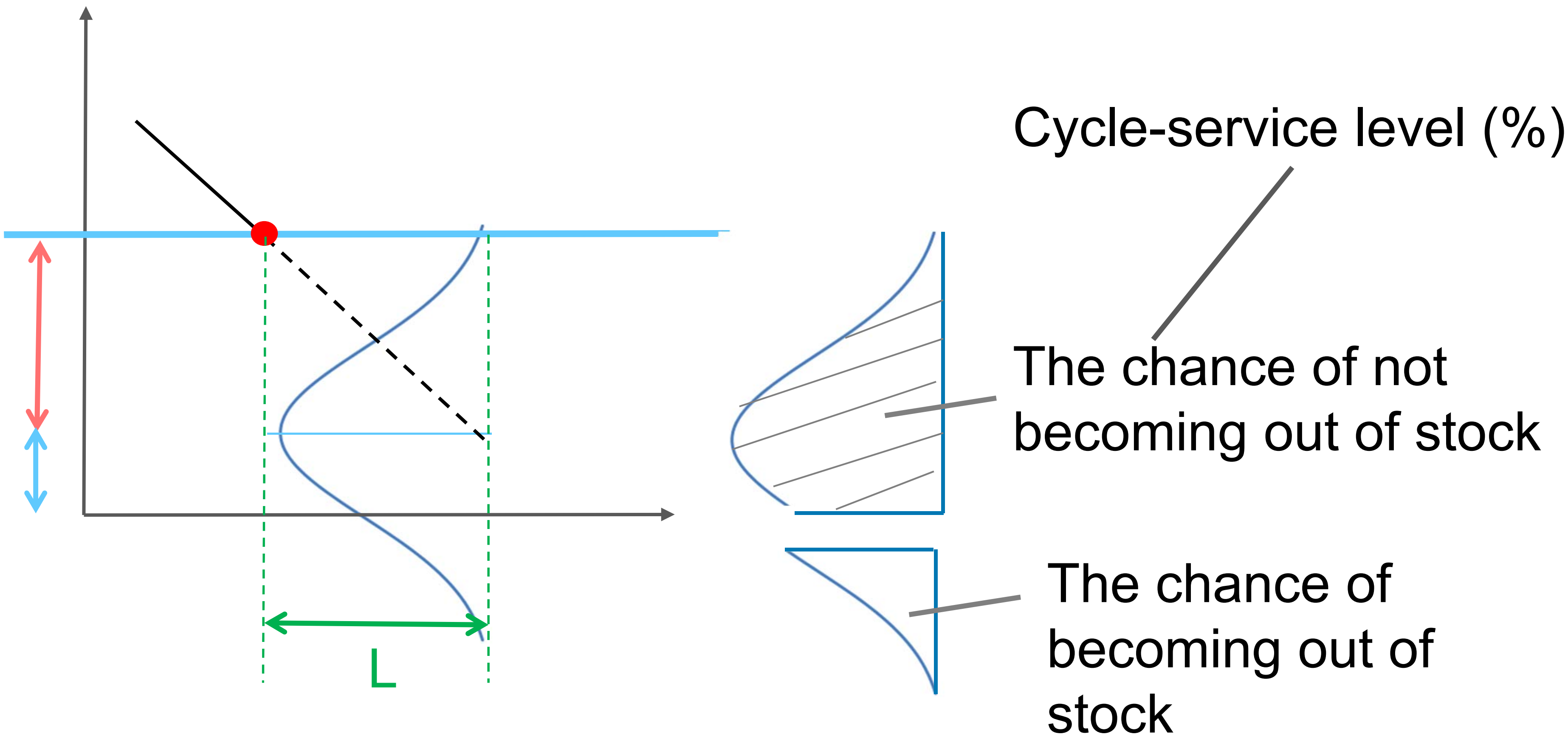
= Average demand during lead time + Safety stock

$$= \bar{d}L$$

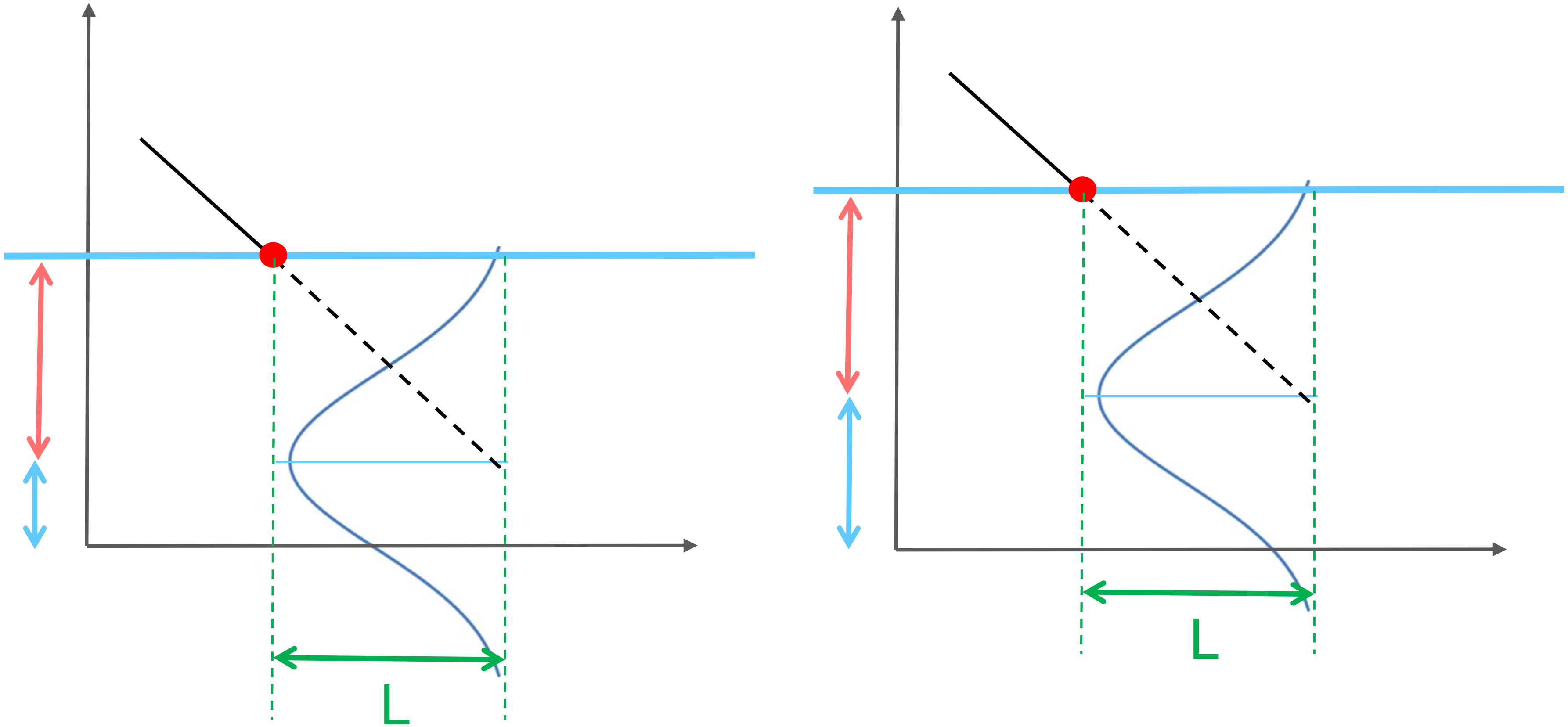
$\bar{d}$: average demand per period (day, week, month)

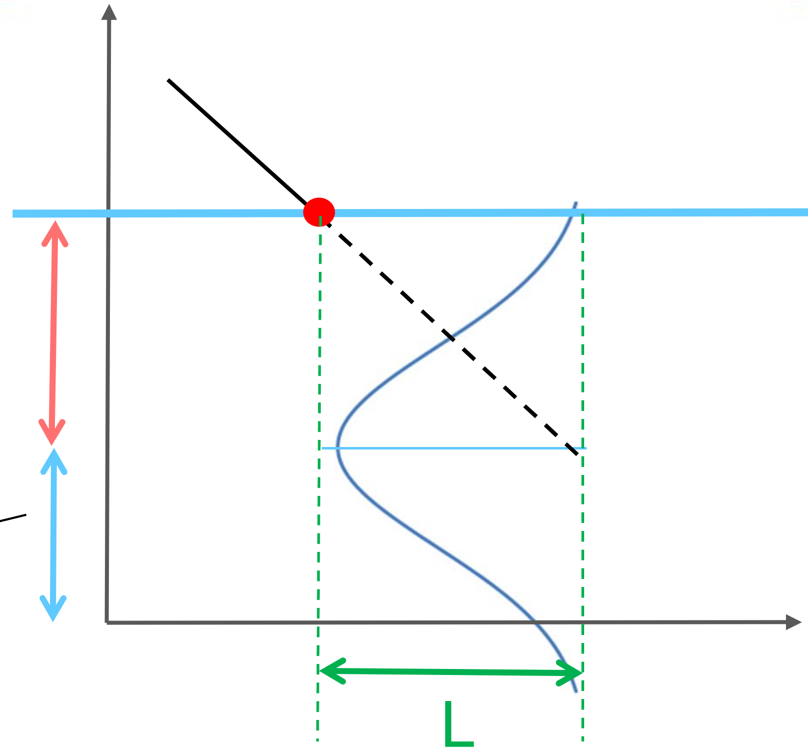
L: Lead time




L
Cycle-service level (%)
The chance of not becoming out of stock
The chance of becoming out of stock

MÄLARDALEN UNIVERSITY
SWEDEN





Safety stock

$= z\ \boldsymbol{\sigma_{dLT}}$

σ_{dLT} : standard deviation of the demand during the lead time

z : how many times of σ_{dLT} needed to achieve the desired cycle-service level

MÄLARDALEN UNIVERSITY
SWEDEN

Re-order point

= Average demand during lead time + Safety stock

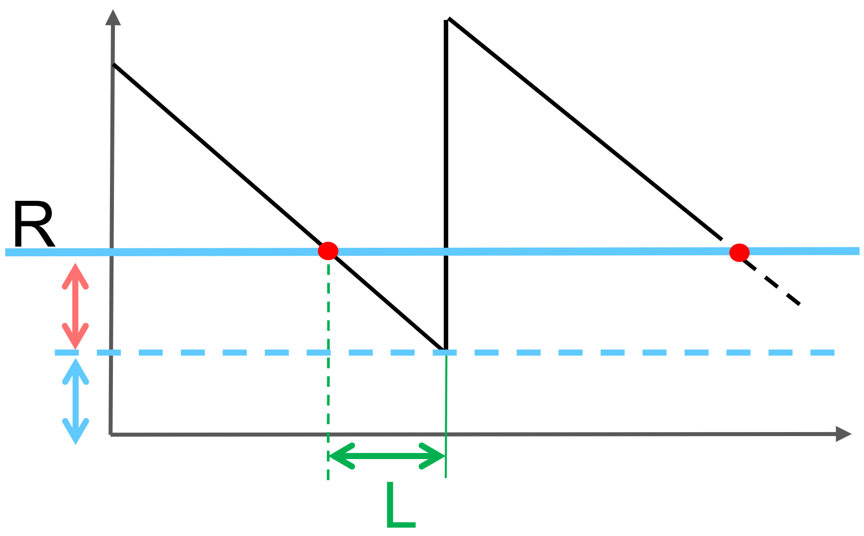
$$\bar{d}L \qquad z\,\sigma_{dLT}$$

$\bar{d}$: average demand per period (day, week, month)

L: Lead time

σ_{dLT} : standard deviation of the demand during the lead time

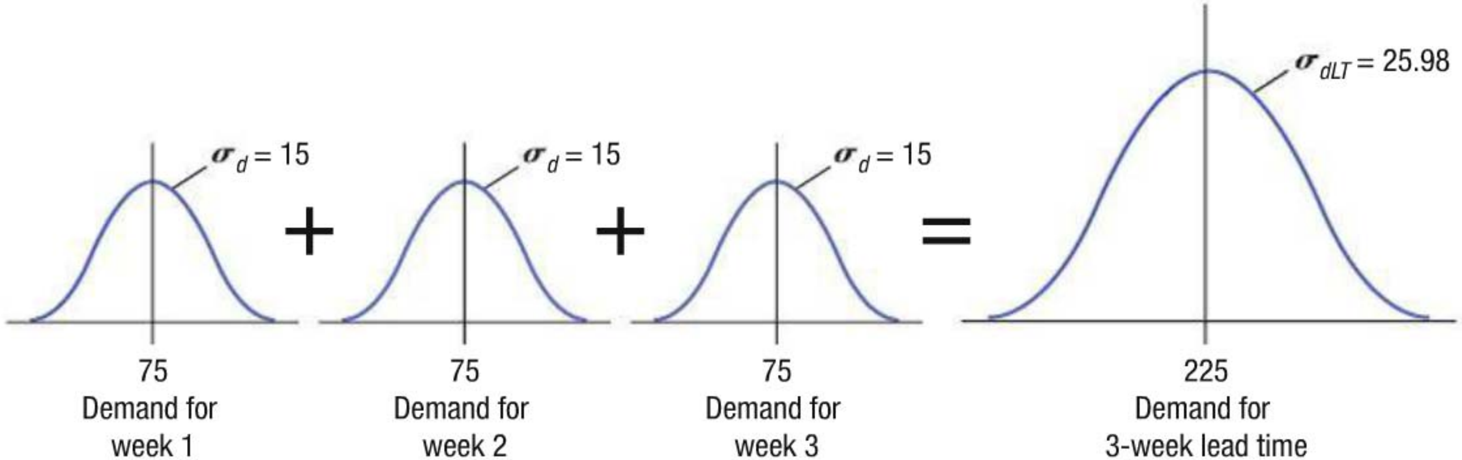
z : Number of σ_{dLT} needed to achieve the cycle-service level



MÄLARDALEN UNIVERSITY
SWEDEN

$$\sigma_d^2 + \sigma_d^2 + \sigma_d^2 + \ldots = \sigma_d^2 L$$

$$\sigma_{dLT} = \sqrt{\sigma_d^2 L} = \sigma_d \sqrt{L}$$



MÄLARDALEN UNIVERSITY SWEDEN

(Example)

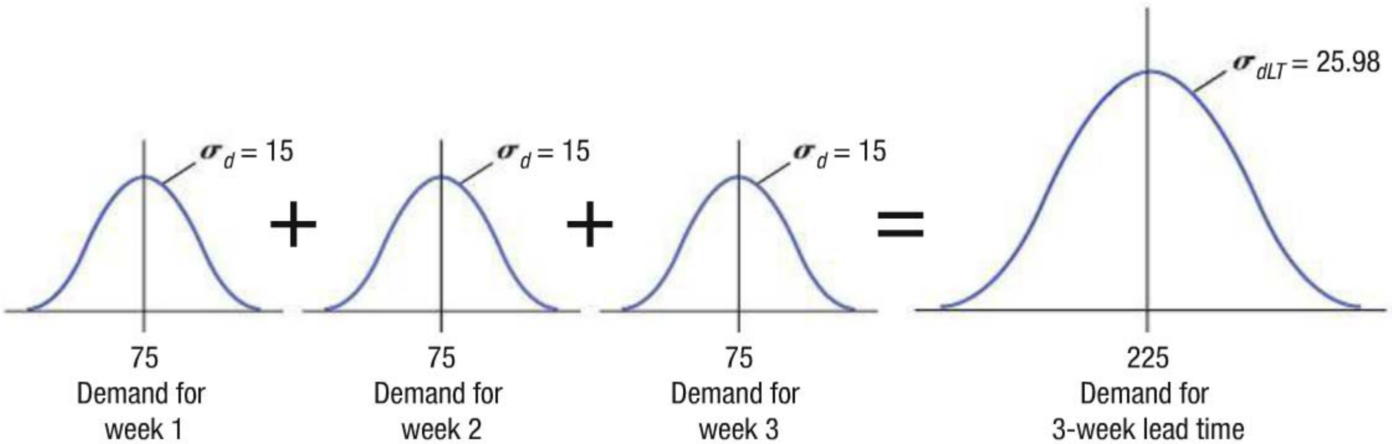
- Weekly average demand 75 with σ_d of 15
- Lead time 3 weeks

$$\sigma_d^2 + \sigma_d^2 + \sigma_d^2 + \ldots = \sigma_d^2 L$$

Variance of demand during 3 weeks = $3\sigma_d^2$

$$\sigma_{dLT} = \sqrt{\sigma_d^2 L} = \sigma_d \sqrt{L}$$

Standard deviation of demand during 3 weeks = $\sqrt{3\sigma_d^2}$ = $\sigma_d\sqrt{3}$



MÄLARDALEN UNIVERSITY
SWEDEN

EXAMPLE 2

Reorder point (ROP) & safety stock in Continuous review system

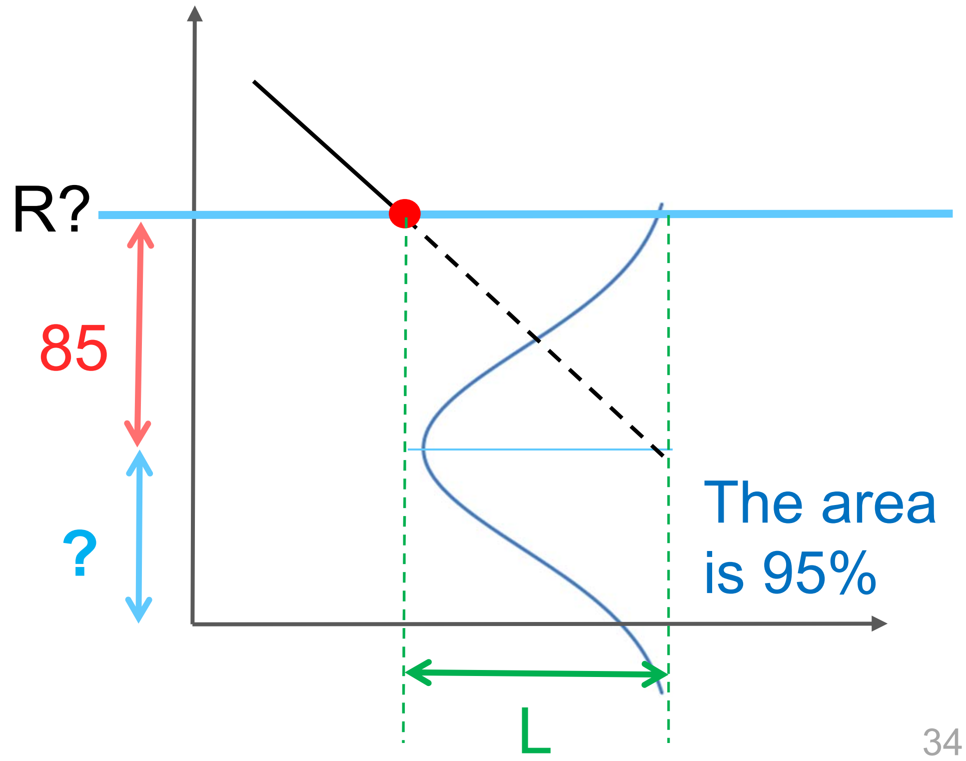
Question

Suppose that the demand during lead time is normally distributed with an average of 85 and $\sigma_{dLT} = 40$. Find the safety stock, and reorder point R, for a 95 percent cycle-service level.

Solution:

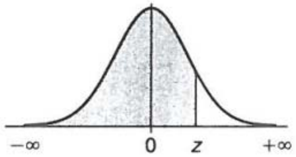
$\bar{d}L = 85$

$R = \bar{d}L + z\,\boldsymbol{\sigma}_{dLT}$



APPENDIX 2

Normal Distribution



	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

z for 95% of area is 1.64 or 1.65

MÄLARDALEN UNIVERSITY SWEDEN

Question

Suppose that the demand during lead time is normally distributed with an average of 85 and $\sigma_{dLT} = 40$. Find the safety stock, and reorder point R, for a 95 percent cycle-service level.

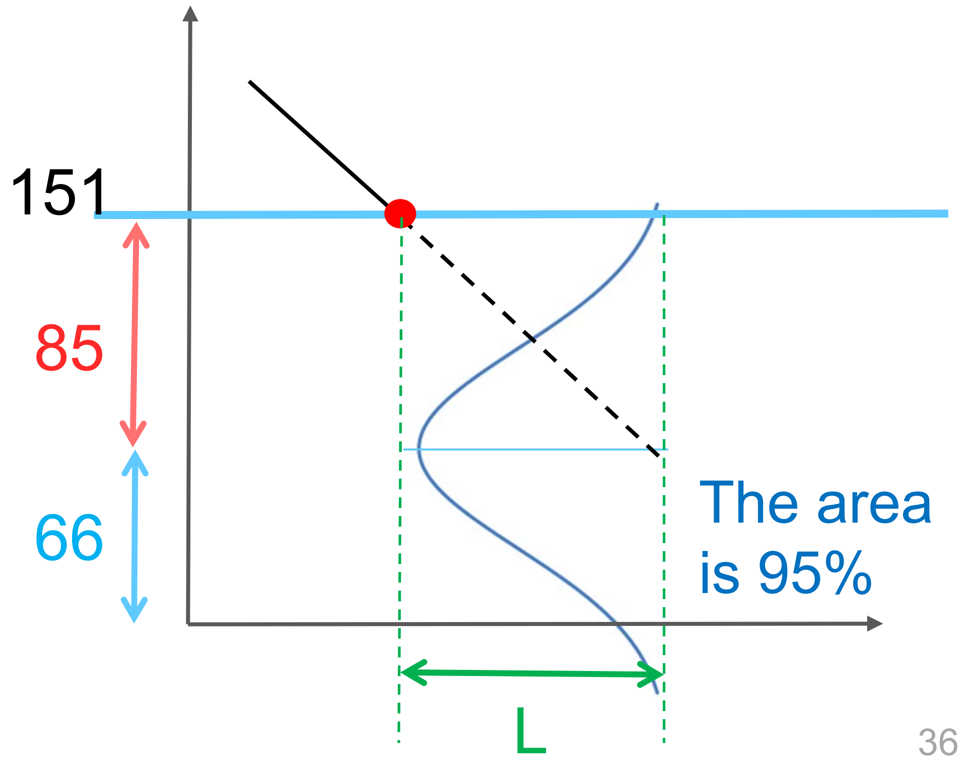
Solution:

$\bar{d}L = 85$

$R = \bar{d}L + z\,\sigma_{dLT}$

$= 85 + 1.65 \times 40$

$= 151$



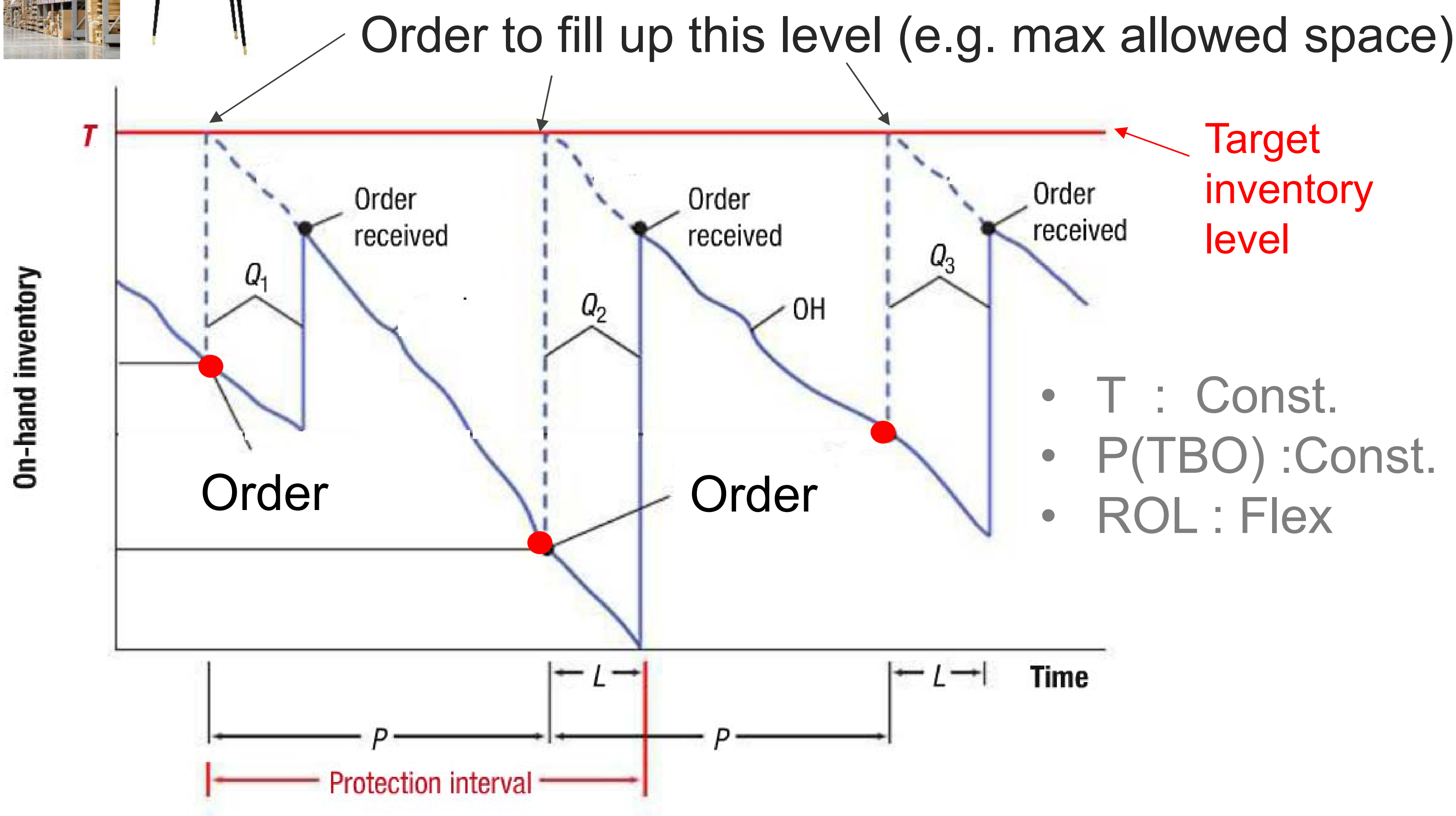
When to place order?

Three well known inventory control systems:

- Continuous review (Q) system
- Periodic review (P) system
- Two bin

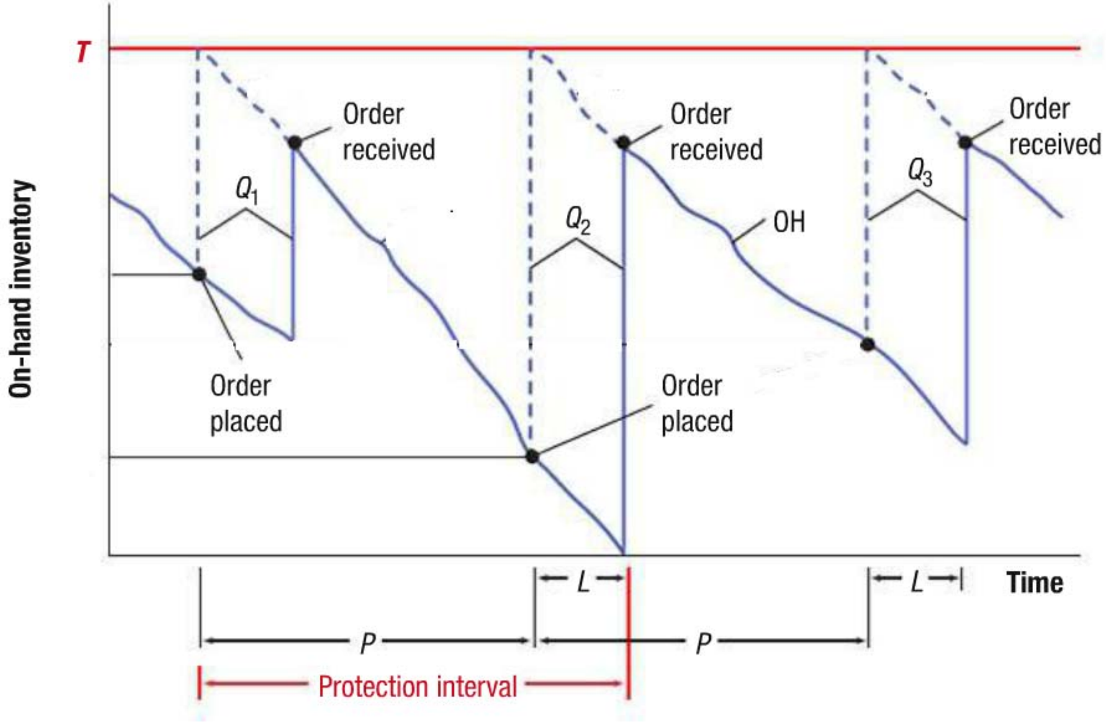
MÄLARDALEN UNIVERSITY SWEDEN

Periodic review (P) system



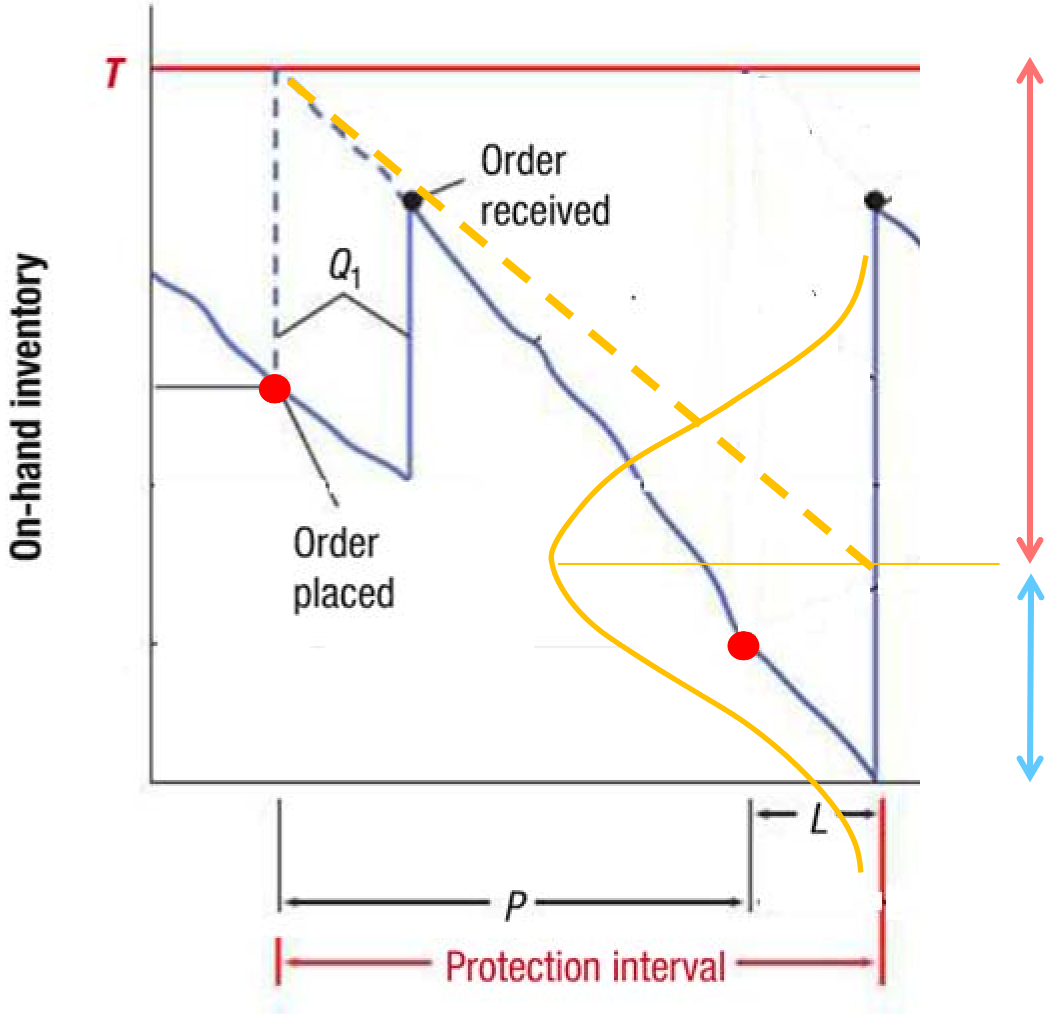
- T : Const.
- P(TBO) :Const.
- ROL : Flex

Periodic review (P) system



- P can be any convenient interval
- P can be based on TBO_{EOQ}
- T is based on demand during P+L

Calculating Target inventory value (T)



Average demand during P+L

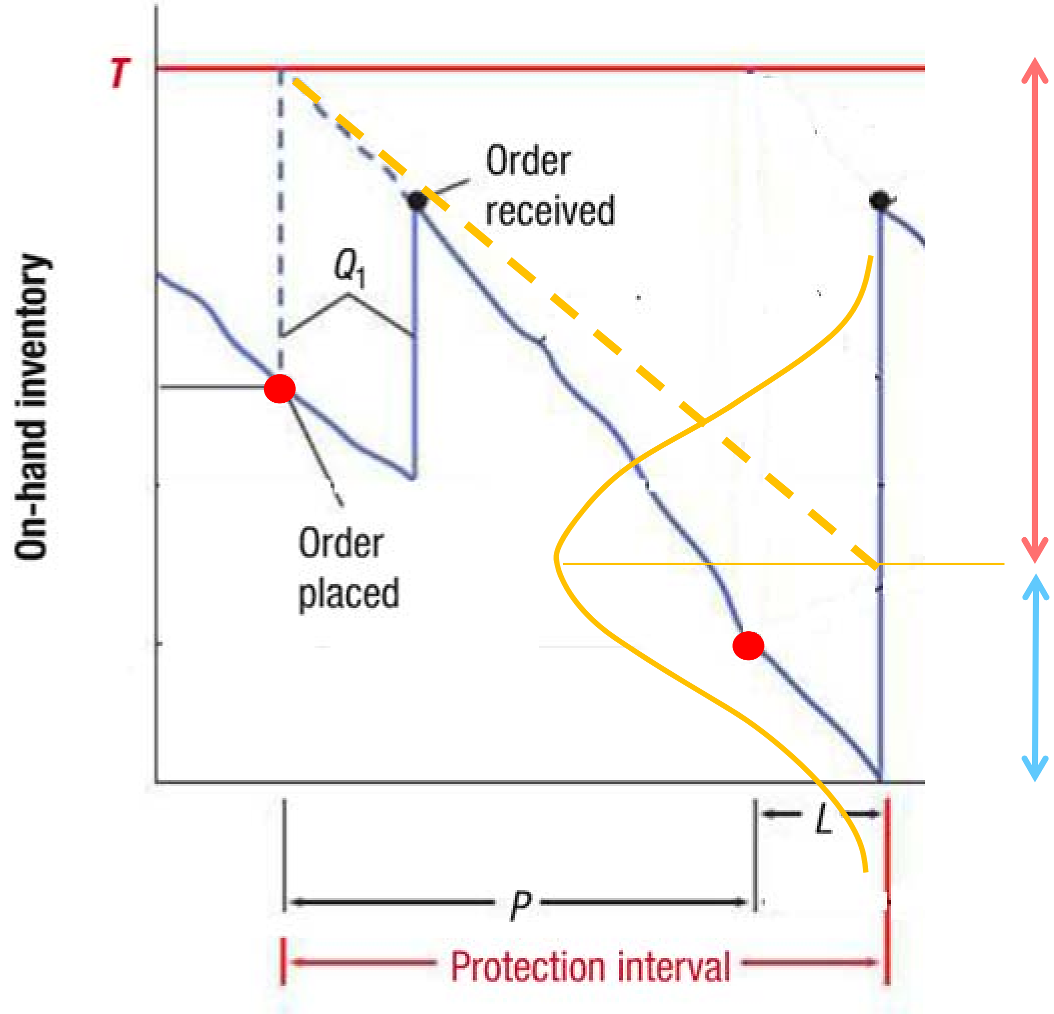
$= \bar{d}\,(P+L)$

Safety stock

$= z\,\sigma_{P+L}$

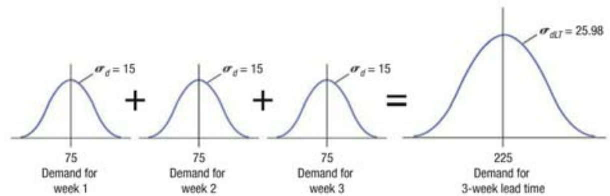
σ_{P+L} : Standard deviation of demand during P+L

Calculating Target inventory value (T)



$$T = \bar{d}\,(P+L) + z\,\boldsymbol{\sigma}_{P+L}$$

$$= \bar{d}\,(P+L) + z\,\boldsymbol{\sigma}_{d}\sqrt{P+L}$$



EXAMPLE 3

P system

MÄLARDALEN UNIVERSITY
SWEDEN

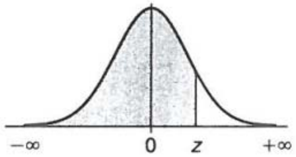
Demand for a bird feeder is normally distributed with a mean of **18 units per week** and a standard deviation in weekly demand of **5 units**. The lead time is **2 weeks**, and the business operates **52 weeks per year**. EOQ is **75 units.** We aim for cycle-service level of **90 percent**.

Find P and T for the P system
Answers are to be rounded to the nearest integer.

$P = TBO_{EOQ} = EOQ/D$ $\quad\quad$ $T = \bar{d}\,(P+L) + z\,\boldsymbol{\sigma_d}\sqrt{P+L}$

APPENDIX 2

Normal Distribution



	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

MÄLARDALEN UNIVERSITY SWEDEN

SOLUTION

We first define D and then P. Here, P is the time between reviews, expressed in weeks because the data are expressed as demand per week:

$$D = (18 \text{ units/week})(52 \text{ weeks/year}) = 936 \text{ units}$$

$$P = \frac{\text{EOQ}}{D}(52) = \frac{75}{936}(52) = 4.2 \text{ or } 4 \text{ weeks}$$

With $\bar{d} = 18$ units per week, an alternative approach is to calculate P by dividing the EOQ by d to get $75/18 \cong 4.2$ or 4 weeks. Either way, we would review the bird feeder inventory every 4 weeks.

We now find the standard deviation of demand over the protection interval $(P + L) = 6$:

$$\sigma_{P+L} = \sigma_d \sqrt{P+L} = 5\sqrt{6} = 12.25 \text{ units}$$

Before calculating T, we also need a z value. For a 90 percent cycle-service level $z = 1.28$. The safety stock becomes

$$\text{Safety stock} = z\sigma_{P+L} = 1.28(12.25) = 15.68 \text{ or } 16 \text{ units}$$

We now solve for T:

T = Average demand during the protection interval + Safety stock

$$= \bar{d}(P + L) + \text{safety stock}$$

$$= (18 \text{ units/week})(6 \text{ weeks}) + 16 \text{ units} = 124 \text{ units}$$

Two bin system

- Order when the other becomes empty

Hybrid systems

Optional replenishment systems

- Optimal review, min-max, or (s,S) system, like the P system
- Reviews IP at fixed time intervals and places a variable-sized order to cover expected needs
- Ensures that a reasonable large order is placed

Base-stock system

- Replenishment order is issued each time a withdrawal is made
- Order quantities vary to keep the inventory position at R
- Minimizes cycle inventory, but increases ordering costs
- Appropriate for expensive items

Visual systems

- Allows employees to place orders when inventory visibly reaches a certain marker. (E.g. Kanban)

MÄLARDALEN UNIVERSITY
SWEDEN

When to place order?

Three well known inventory control systems:

- Continuous review (Q) system
- Periodic review (P) system
- Two bin